Original Article

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EIGENVECTOR SELECTION INDICES FOR IMPROVING MILK YIELD AND PERSISTENCY IN EGYPTIAN BUFFALO

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ABSTRACT

The genetic gains were estimated for milk production and persistency, derived from random regression models, using eigenvector indices, and they were compared with the traditional selection index. The data set contained 4971 test day milk yield recorded for 691 buffalo cows, daughters of 120 sires and 532 dams. The model included the random effects of direct additive genetic, permanent environment and error, whereas the fixed effects were herd test day, year and season of calving and parity, and as a covariate, it was milk days. The first and the 2nd eigenvalues explained 73.1 and 22.9% of the variation of the random regression coefficients, respectively, suggesting that the use of the first two eigenvectors is sufficient. Genetic responses in total milk yield (TMY) based on the first eigenvector index (I_{e1}) and that based on the conventional selection (I_{MY}) have close gain of about 171 kg in each index. The second eigenvector index (I_{e2}) showed an increase in TMY (9.91 kg), and thus an increase in the persistency (0.86 kg). The TMY and persistency are the two economically important traits in dairy production, additional genetic gains in persistency and high genetic gain for TMY could be obtained using the 2nd eigenvector index (I^*_2).

Keywords: *Bubalus bubalis*, buffaloes, eigenvalue, eigenvector selection indices, milk yield, persistency, Egyptian buffalo

INTORDUCTION

When the purpose of a breeding program is to improve multiple traits, the most efficient way to use the available information is usually to construct a selection index (Hazel, 1943). Selection for lactation curve parameters especially persistency and milk yield would result in improving milk yield (Dekkers *et al*., 1997; Swalve, 2000).

The selection indices for improving milk production and persistency were derived from the decomposing the covariance function into its eigenvalues and eigenfunctions and eigenvectors of the additive genetic coefficients of random regression models (RRM) as reported by Togashi and Lin (2006); Savegnago *et al*. (2013) in cattle and Flores and Van der Werf (2014) in buffalo. Each eigenfunction represents the curve pattern

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of the longitudinal trait population mean curve in each dimension of the matrix of additive RRM genetic coefficients (Kirkpatrick and Heckman, 1989). There are a few studies related with the use of eigenvectors as a selection index (SI), Togashi and Lin (2006); Savegnago *et al*. (2013) in cattle and Flores and Van der Werf (2014) in buffalo explained that the various eigenvector indices are based on the K matrix estimated from test day records. The first (major) eigenvector index produced a constant response for each day of lactation. Daily genetic responses to the second eigenvector index were associated with increased genetic gain of persistence. Genetic response to the third eigenvector index was less important unlike Togashi and Lin (2006). The genetic response to the 4^{th} eigenvector index remained near zero during lactation. In dairy buffaloes, studies regarding lactation persistency measures and their relationship with other traits have been limited. Geetha *et al*. (2006) used random regression to estimate EBVs for daily yields and used these EBVs to derive several persistency measures.

The objective of the present study, therefore, was to estimate population genetic parameters and trends for reproductive performance in Egyptian buffalo.

The objectives of the present study, therefore, was to (1) The eigenvalues to decompose, (2) To derive the selection genetic responses in lactation based on individual eigenvector indices, (3) To construct the sequential eigenvector indices for milk yield and persistency, (4) To calculate the selection responses in total milk yield using sequential eigenvector indices, (5) To estimate the selection response in total milk yield and persistency using sequential eigenvector indices, (6) To estimate the SI based on the lactation breeding values (EBV) until 301 DIM.

MATERIALS AND METHODS

Data were collected monthly from four test buffalo herds (El-Nattafe El-Gadid, El-Nattafe El-Kadim, Mahalet Mousa and El-Gemmiza), Animal Production Research Institute (APRI), from 1999 to 2009. It consisted of 4971 test-day records. TD records for milk yield were measured following an alternative am-pm monthly recording scheme. Milking was performed twice daily at 7am and 4pm during the lactation period. The structure of the analyzed data is shown in Table 1.

Estimation of relative economic values for milk yield and persistency

The major concepts of persistency measures are based on mathematical lactation curve models described by Dekkers *et al*. (1996) as follows:

$$P = 110\ [(\mu_{60}-\mu_{280})-(Y_{60}-Y_{280})],$$

Where μ_i and Y_i are population average and individual yield at i^{th} DIM of 60 and 280 days, respectively. Persistence (P) defines the area of the triangle representing different performance between days 60 and 280 of persistent lactation compared to the mean shape lactation curve (Swalve, 1995). Then, milk yield traits per lactation were estimated and adjusted to 301 days in milk (DIM) using Fleischmann's method as cited by El-Saied *et al*. (1999).

Costs and returns are calculated based on actual phenotypic performance. The total annual profit for the herd was derived from the difference between system costs and income. In this study, all costs and prices are expressed based on marketable product averages and Egyptian dairying (APRI) costs. The unit of production is a buffalo herd, and the unit of time is the year. Inputs to production were food, administration, buildings, maintenance, water, electricity, medicines, and wages. Costs were calculated based on cost per kilogram of milk. Profits were therefore derived from the difference between revenue (R) and cost (C) for buffaloes per year. The sum of these costs accounted for the fixed costs per animal per day was 3.15 L.E. To get the profit, we had to calculate the difference between revenue (R) which was 4 L.E. per unit and costs (C). The net profit was 0.85 L.E.

The calculation of the persistency equation was 31933 kg, but the calculation of total milk yield was 1289 kg as reported by Fleischmann's method. Therefore, the relative economic value between total milk yield and persistency was 1:25. This ratio (1:25) is calculated by dividing 31933 to 1289.

Legendre polynomial functions

Kirkpatrick *et al.* (1990) proposed a selection index (SI) based on the Eigen-analysis method and all the indices derived were based on the genetic measurements from the RRM. Eigenvectors of the additive genetic coefficient matrix were used to construct SIs and to assess their effects on milk yield and persistency. The eigenvectors are orthogonal, and this property can be used to analyze each eigenfunction independently (individually) or combined (sequentially). The K matrix used in this study was estimated using the Legendre cubic polynomial ($k = 3$) under the test day RR animal model. A cubic legendre polynomial resulted in four eigenvectors and thus, four-index traits.

In matrix notation, the orthogonal legendre polynomials can be written as $\Phi = M\Lambda$, in which M is the matrix with the polynomial values for standardized units of time (wt) of order (t by k), in which t is the DIM, k is the number of parameters of the polynomial, Λ is the matrix with the coefficients of the standardized legendre polynomials, and Φ is a legendre polynomial coefficients from DIM = 4 through 304 (Mrode, 2005). The additive genetic and the non-genetic animal (co)variance matrices for monthly test day milk yield ($\hat{G}$ and $\hat{C}$, respectively) were calculated by pre- and post-multiplying the K_a and K_c matrices by Φ, resulting in $\hat{G} = \Phi K_a \Phi'$ and $\hat{C} = \Phi K_c \Phi'$, in which Φ` is the transpose of Φ.

The covariance matrix for the additive genetic coefficients of the third order legendre polynomials (K) was as follows:

$$K = \begin{bmatrix} 0.65218 & 0.00143 & -0.12748 & 0.17008 \\ 0.00143 & 0.11337 & 0.10630 & 0.02239 \\ -0.12748 & 0.10630 & 0.13397 & 0.00719 \\ 0.17008 & 0.02239 & 0.00719 & 0.09272 \end{bmatrix}$$

Constructing selection index based on eigenvalues decompositions (Eigenfunction)

The SAS/IML model was used to compute eigenvalues (EV) and eigenvectors (E) for covariance matrices of regression coefficients to quantify the contributions. Let $\alpha = [\alpha_0\ \alpha_1\ .\ .\ .\ \alpha_{k-1}]$` be a ($k \times 1$) vector of the additive genetic random regression (RR) coefficients for each animal by fitting a Legendre polynomial of degree (k-1). The variance of vector α is a ($k \times k$) additive genetic RR covariance matrix (K) with k pairs of eigenvalues (EV_i) and normalized (orthogonal) eigenvectors (E) (e_i, i = 1, 2, ..., k). Let E be a ($k \times k$) matrix containing these orthogonal eigenvectors as columns. The genetic covariance matrix (G) for daily lactation performance from DIM = 4 to 304 days of lactation is $G = \Phi K \Phi'$ where Φ is a ($301 \times k$) matrix of legendre polynomial coefficients (i.e. covariates) evaluated from DIM = 4 through 304 days.

The selection index (I_K) built on the eigenvectors of K are defined as:

$$I_k = \sum_{i=1}^{k} b_i \left(\alpha' e_i\right) = b' E' \alpha$$

Where b is a (k×1) vector of selection index coefficients. According to this definition, an "index trait" ($\alpha' e_i$) are linear combinations of the element-weighted additive genetic RR coefficients of a given eigenvector, thus resulting in a total of k index traits. The first index feature corresponds to the first eigenvector with the largest eigenvalue. The second index trait corresponds to the second eigenvector with the second largest eigenvalue, and so on. Statistical, these "synthetic" index traits are the principal components. The variance of index I_K is calculated as:

$$\sigma^2_{I_K} = b' E' K E b = b' D b$$

Where D is a diagonal matrix with the eigenvalues of K on the diagonal elements (Searle, 1966), indicating that the index traits are uncorrelated. The total variance of this index trait is:

$$Var\left(1' E' \alpha\right) = 1' D 1 = \sum_{i=1}^{K} EV_i$$

Where 1 is the sum vector of order k and EV_i being the ith eigenvalue of K.

Constructing individual eigenvector selection indices

The individual eigenvector indices were calculated as:

$$I_j = e'_j \alpha_i,$$

Where I_j is the j^{th} individual selection index based on the j^{th} eigenvector; e'_j is a line vector of the j^{th} eigenvector of order 1 by k; and α_i is the column vector of order k by 1 for the solution of the additive genetic RR coefficients of the i^{th} animal, fitted by legendre polynomial of degree (k-1). Each index trait was used separately as a selection criterion ($I_{jth} = e_j` \alpha_i = x_i$, where j^{th} = first, second, third, or fourth eigenvector) to assess the impact of the j^{th} eigenvector on milk yield and persistency.

To characterize each eigenvector of K, genetic responses associated with individual eigenvectors were calculated. The genetic selection response based on I_j from each DIM (Δ); that is, 4 to 304 DIM is defined as:

$$\Delta = \Phi K_a e_j \left[\frac{i}{ó_{I_j}}\right]$$

Where $\Delta = [\Delta G_1 \ \Delta G_5 \ \ldots \ \Delta G_{301}]'$ is a transpose vector for the genetic gain in milk production in each DIM; i is the selection intensity = 1; $ó^2_{I_j} = e`_j K e_j = EV_j$ is the variance of the j^{th} index; and EV_j is the j^{th} eigenvalue of K.

Constructing sequential eigenvector selection indices

The sequential eigenvector indices were calculated as: $I_e = b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4$; $x_i = e'_j \alpha_i$, and x from 1 to 4.

The full and reduced indices were calculated according to the vector of index coefficients (b) as:

$$b = D^{-1} E` K \Phi` 1$$

Index traits are orthogonal, the index coefficient for a given index trait is the same across these consecutive indexes (e.g., b_2 for the second index trait in I_{e2}, I_{e3}, and I_{e4} are identical), where I_e is the selection index based on all eigenvectors; e'_j is a line vector of the j^{th} eigenvector of order 1 by k; and α_i is the column vector of order k by 1 for the solution of the additive genetic RR coefficients of the i^{th} animal, fitted by Legendre polynomial of degree (k-1). The full eigenvector index denoted by I_{e4} consists of 4 index traits. The last index trait of the full eigenvector index was sequentially removed to yield 3 reduced indices I_{e3}, I_{e2}, and I_{e1} where the subscripted number indicates the number of index traits included in an index.

The genetic responses (Δ) in daily lactation yields based on sequential eigenvector index from DIM = 4 to 304 days are calculated as:

$$\Delta = \Phi K E b \left[\frac{i}{\sigma_{I_j}} \right]$$

Where $\sigma^2_I = b`Db$, as defined previously.

Constructing sequential eigenvector selection indices using the relative economic weights

Let g_{60} and g_{280} to be the genetic values at 60 and 280 DIM, respectively, a_1 and a_2 to be the economic weights of milk yield and persistency. The eigenvector index (I*) for maximizing the net merit of the linear combination of milk yield and persistency is defined as:

$$I^* = b' E' \alpha,$$

where b is a vector 1 by k for the coefficients of the selection index and E` is the transposed matrix of the eigenvectors of the matrix K. A persistency measure is defined as the difference between g_{280} and g_{60}.

The selection index coefficients were calculated as follows:

$$b = D^{-1} E' K (a_1 \Phi^{*\prime} 1 + a_2 (\Phi'_{280} - \Phi'_{60})).$$

Where D^{-1} is the inverse of the diagonal eigenvalues of the matrix K; 1 is the vector of ones of appropriate dimensions; $\Phi^{*\prime}$ is a (k by 299) matrix obtained by deleting the rows of Φ corresponding to 60 and 280 DIM; and a_1 and a_2 are the relative economic weights for milk yield and persistency, respectively. The exclusion of $\Phi`_{60}$ and $\Phi`_{280}$ from $\Phi^{*\prime}$ was done to avoid the duplication of the information of persistency on milk yield in the net merit.

The genetic responses (Δ) in daily lactation yields based on sequential selection index and accounting for the relative economic values (a_i) from DIM = 4 to 304 days are computed as follows:

$$\Delta = \Phi K E b \left[\frac{i}{\sigma^*_I} \right]$$

Where $\sigma^{2*}_I = b' E' K E b = b' D b$, as defined previously.

Fitting the cubic legendre polynomial (k = 3), the full index (I^*_4) that consists of 4 index traits was derived from the 4 eigenvectors. Dropping the last index trait sequentially, the reduced indices of I^*_3, I^*_2, and I^*_1 were produced. The selection intensity was set to be 1.0 for all selection criteria. The genetic responses from different indices were computed and they were used for direct comparisons.

Constructing the selection indices based on lactation EBV (Traditional selection indices)

The traditional index did not use the eigenvector decomposition of K. The traditional selection index based on estimated the breeding values (EBV) for milk yield until 301 DIM (I_{MY301}) was given by I_{MY301} = 1` Φ α, and the genetic gains in each DIM were calculated by $\Delta = i\sqrt{1`G1}$, using the random regression model (RRM).

Calculating the genetic responses for eigenvector and traditional selection indices

The selection response from 4 to 304 DIM based on individual or sequential eigenvector indices were compared with the selection response from the traditional index for milk production and persistency. The selection responses of the persistency (ΔG_P) using the individual, sequential, and traditional indices were calculated as the differences between the genetic gains for 280 DIM minus 60 DIM (ΔG_{280} - ΔG_{60}). The selection responses of total milk yield from 4 to 304 DIM (ΔG_{MY}) were calculated by the summation of genetic gains from each DIM. The selection intensity (i) was set as 1 in all selection indices in this study.

RESULTS AND DISCUSSIONS

Eigenvalues decomposition (Eigenfunction)

The eigenvalues (EV_j) and eigenvectors (E) of the additive genetic RR coefficient matrix (K) using legendre cubic polynomials are shown in Table 2.

In this table, the first and the 2nd eigenvalues explained 73.1 and 22.9% of the variation of the random regression coefficients, respectively, whereas the 3rd and 4th eigenvalues accounted for a combined total of only 4%. These values are similar to those values obtained for the buffalo in Philippine (Flores and Van der Werf, 2014) and lower than the values for Holstein cows in Japan (Togashi and Lin, 2006) and Brazil (Savegnago *et al*., 2013).

The first element in the leading eigenvector is the largest (Table 2), suggesting that the first eigenvector (constant = 0.95) contributes significantly to lactation variation. The remaining three eigenvectors have the largest elements associated with linear (-0.02), quadratic (-0.21) and cubic (0.25) parts of the curve, as stated by many researchers, affects the variability of the lactation curve in different ways. (Kirkpatric *et al*., 1990; Olori *et al*., 1999; Togashi and Lin, 2006; Savegnago *et al*., 2013; Flores and Van der Werf, 2014).

Using of the first two eigenvalues explained 96.0% of the variation in the breeding goal of improving the milk yield and persistency as opposed to 99.99% of the first three eigenvalues (Table 2), suggesting that the use of the first two eigenvalues is sufficient. It is therefore worth noting that it is applicable to improve milk yield and persistence using the first three eigenvalues (Savegnago *et al*., 2013; Flores and Van der Werf, 2014). The total contribution of the fourth eigenvalues to the breeding target is negligible (Togashi and Lin, 2006). In this regard, Druet *et al*. (2003) suggested that using the reduced set of eigenvectors for genetic evaluation reduces computational cost.

The eigenfunction related to the first eigenvalue (EV_1) was positive and constant throughout the lactation period as shown in Figure 1. The result suggests that most of the variation in test day milk yield is explained by genetic component acting almost constantly throughout the lactation period. A major part of the observed genetic variance can be explained by a factor which is practically constant throughout the lactation, suggesting that if selection was based on any test day milk yield (TDMY), a genetic gain would be obtained for all periods.

The eigenfunction related to the 2[nd] eigenvalue (EV2) was negative in the first half of lactation period but became positive from 133 to 278 DIM (Figure 1), where it converted to be a negative till the end of DIM. This eigenvalue may correspond to a genetic component for persistency and indicates that it could be possible to select for the persistency in lactation (Flores and Van der Werf, 2014).

The third eigenvalue (EV_3) is very low and the eigenfunction related to EV_3 was negative up to 76 DIM and positive after that but converted to be a negative again from 119 to 246 DIM (Figure 1), where it became positive to the end of DIM.

The fourth eigenvalue (EV_4) is close to zero and the eigenfunction related to EV_4 was positive up to 52 DIM and negative after that but converted to be a positive again from 197 to the end of DIM (Figure 1). The fourth eigenvalue didn't affect the trajectory of the lactation milk curve as stated by Togashi and Lin (2006).

Constructed individual eigenvector indices

Four individual eigenvector indices (I_{e1}, I_{e2}, I_{e3} and I_{e4}) constructed are given in Table (3) and they were compared with the conventional selection in terms of genetic responses for milk yield and persistency.

Genetic responses in total milk yield based on the first eigenvector index (I_{e1}) and that based on the conventional selection (I_{MY}) have close gain of about 171 kg in each index (Table 3) and the two indices (the first eigenvector index and the conventional selection index) have the same trend as shown in Figure 2. The second eigenvector index (I_{e2}) showed an increase in total milk yield (9.91 kg, Table 3) a small increase and DIM 280 a significant increase, and thus an increase in the persistency (0.86 kg) but decreasing DIM 60 (peak yield), these results agreed with Togashi and Lin (2006); Savegnago *et al.* (2013). The Third eigenvector index (I_{e3}) decreased milk yield (-9.36 kg, Table 3), DIM 280 and persistency but increased a small amount in the DIM 60. The fourth eigenvector index (I_{e4}) showed an increase with small amount in milk yield (0.15 kg, Table 3), DIM 60 and 280, but with decrease in persistency. Genetic response in the persistency was positive just in second eigenvector index (0.86 kg), but in the other eigenvectors indices they were negative as stated by Togashi and Lin (2006).

In Figure 2, the genetic responses in daily milk using the I_{e1} was increased from -0.082 kg at the first DIM to be 0.83 kg at 103 DIM and then reduced at 0.398 kg at 243 DIM and increased to be 0.667 kg at 301 DIM. I_{e1} shows a near-horizontal pattern, indicating that it is primarily responsible for the constant increase in daily milk volume along lactation (Olori *et al*., 1999; Togashi and Lin, 2006; Savegnago *et al*., 2013; Flores and Van der Werf, 2014). Accordingly, selecting the animals based on the I_{e1} would change mainly the buffalo cow's milk yield (Togashi and Lin, 2006; Savegnago *et al*., 2013; Flores and Van der Werf, 2014).

Genetic response in daily milk yield from using the second eigenvector index (I_{e2}) was almost a horizontal pattern of -0.1 kg until DIM 63 day but then it reduced to be -0.22 Kg in the 156 DIM and increased thereafter to the end of DIM to be 1.18 kg (Figure 2). These results suggest that I_{e2} is strongly associated with changes in the lactation curve, because it has greater eigenvalues than the 3[rd] and 4[th] indices (I_{e3} and I_{e4}) and the only increase was recorded in the persistency (ΔG_p) to be 0.86 kg (Table 3). Thus, selection based on the I_{e2} could result in genetic gains in the end of the lactation and persistency and would also result in positive genetic gain in total milk yield as reported by Togashi and Lin (2006); Savegnago *et al*. (2013).

Using the I_{e3} (Figure 2), the genetic responses in daily milk yield were negative before 24 DIM, positive between 25 to 138 DIM, and negative from 139 to 280 DIM, then positive until the end of the lactation, forming a concave curve (Figure 2). The I_{e3} was less important, because the genetic gain for milk yield and persistency were negative (Savegnago *et al*., 2013). Genetic responses in daily milk yield recorded by the I_{e4} was almost around zero across the lactation period (Figure 2), giving a combined gain of 0.15 kg in total milk yield (Table 3). The I_{e4} was the least important index because the genetic gain for milk yield was very low and persistency was negative (Togashi and Lin, 2006). Therefore, the combined results of both the I_{e3} and I_{e4} indices play only a subordinate role in genetic improvement of milk yield and persistency.

Constructed sequential eigenvector indices

The four consecutive eigenvector indices constructed are shown in Table 4 and the genetic responses (kg) at these series of sequential eigenvector indices were calculated to maximize the total milk yield and persistency.

These results showed that the status of persistency does not exist in all indices constructed especially with the traditional index. The second eigenvector index (I^*_2) is the best with the persistency, although it produces a positive value in genetic gain of persistency. This means that selecting for higher milk yield alone; persistency will deteriorate because of the high correlation between total milk yield and peak milk yield (Flores and Van der Werf, 2014).

The alternative selection criterion to increase milk yield or minimizing the deterioration in persistency is to reduce the response of selection in milk yield. A 5% reduction in response compared to the selection for milk yield alone is fine, while having an increase in persistency response (Flores and Van der Werf, 2014), but in our cases the reduction is 1% when we choose I^*_2. So, the significant improvement in milk production and the reduction of deterioration in the persistency provides even a slight improvement in the persistency. The choice of I^*_2 is correct since the slight reduction in response of milk yield is compensated by the improving of persistency. In calculation of economic index, it is more profitable to improve milk yield in the second half of the lactation period due to less feed cost.

Since the genetic gains in milk yield, when using sequential eigenvector indices were since can to that obtained when using the traditional selection index (about 171 kg, Table 4). Therefore, traditional index could be used because it is simple method to calculate as reported by many investigators (Togashi and Lin, 2004; Geetha *et al*., 2006; Aspilcueta-Borquis *et al*., 2012; Savegnago *et al*., 2013). When the persistency is defined as (EBV at 280 DIM - EBV at 60 DIM), it will show a certain genetic gain even if the breeding goal is manly for milk production alone (I_{MY301}) as shown in Table 4.

When aiming to maximize milk yield alone, the index's coefficient for the second eigenvector (b_2) was 20.80 kg (Table 4). In contrast, with the goal of maximizing milk yield and persistency, b_2 was increased to be 64.41 kg (a_1:a_2 = 1:25, Table 4). The index's coefficient for the third eigenvector (b_3) was -46.72 kg when the selection was directed for milk yield alone but it was decreased to be -65.79 kg when selection was for milk yield and persistency with a_1:a_2 = 1:25. The same trend was observed in the index coefficient

Table 1. Data structure of test day of lactation (TD) used in analysis of Egyptian buffalo records.

Item	**No.**
No. of sires	120
No. of dams	532
No. of cows with records	691
No. of base animals	469
No of non-base animals	684
Total number of animals	1153
Total number of lactation records	4971

Table 2. Eigenvalues decomposed (with the proportion relative to the total variation) and the eigenvectors of the additive genetic covariance matrix (k) for lactation yield.

	1st	**2nd**	**3rd**	**4th**
Eigenvalues decomposed (x_j)	0.72	0.23	0.04	0.000009
	(73.1)	(22.9)	(4.04)	(0.0009)
Eigenvectors of legendre polynomial functions (e_i)				
Constant part of the curve	0.95	0.09	-0.22	0.22
Linear part of the curve	-0.02	0.68	-0.42	-0.60
Quadratic part of the curve	-0.21	0.67	0.11	0.70
Cubic part of the curve	0.25	0.26	0.87	-0.31

Table 3. The individual eigenvector indices constructed and the genetic responses in total milk yield (kg) in comparison with the conventional selection index (I_{MY}).

Individual eigenvector indices constructed	**Genetic gain in lactation yield (kg)**			
	ΔG_{MY}*	**ΔG_P**	**ΔG_{60}**	**ΔG_{280}**
$I_{e1} = x_1$	171.07	-0.21	0.71	0.50
$I_{e2} = x_2$	9.91	0.86	-0.10	0.75
$I_{e3} = x_3$	-9.36	-0.15	0.15	-0.0001
$I_{e4} = x_4$	0.15	-0.001	0.001	0.0001
conventional selection index (I_{MY})	171.61	-0.15	0.69	0.54

*ΔG_{MY} = Genetic gain in total milk yield; $\Delta G_P = \Delta G_{280} - \Delta G_{60}$.

Table 4. Sequential eigenvector selection indices constructed and the genetic responses (kg) when the net merit consists of milk yield alone or as a combination of milk yield and persistency.

Selection indices	ΔG_{MY}	ΔG_P	ΔG_{60}	ΔG_{280}
Sequential eigenvector selection indices for improving milk yield alone				
$I_1 = 200.91X_1$	171.34	0	0.569	0.569
$I_2 = 200.91X_1+20.80X_2$	171.60	-0.005	0.572	0.567
$I_3 = 200.91X_1+20.80X_2-46.72X_3$	171.61	-0.133	0.556	0.423
$I_4 = 200.91X_1+20.80X_2-46.72X_3+48.42X_4$	171.61	-0.154	0.695	0.541
Sequential eigenvector selection indices for improving milk yield and persistency (with a_1:a_2 = 1:25)				
$I^*_1 = 193.31X_1$	170.20	0	0.569	0.569
$I^*_2 = 193.31X_1+64.41X_2$	169.84	0.045	0.550	0.595
$I^*_3 = 193.31X_1+64.41X_2-65.79X_3$	169.15	-0.026	0.525	0.499
$I^*_4 = 193.31X_1+64.41X_2-65.79X_3+37.16X_4$	168.94	-0.028	0.673	0.644
Traditional selection index				
I_{MY301}	171.61	-1.283	10.920	9.637

$\Delta G_P = \Delta G_{280} - \Delta G_{60}$; a_1 and a_2 are the relative economic weights for milk yield and persistency, respectively.

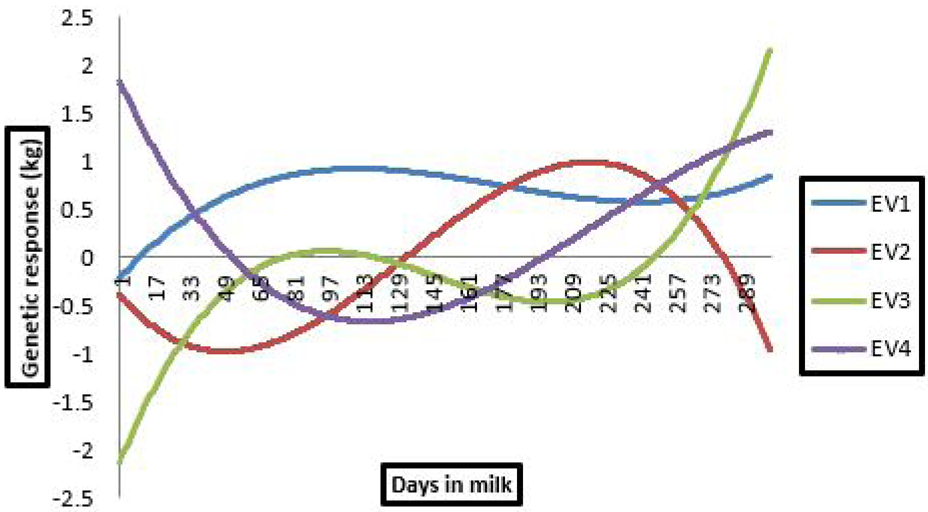

Figure 1. Genetic selection responses in daily milk yield (kg) related to the four eigenvalues of the genetic covariance matrix.

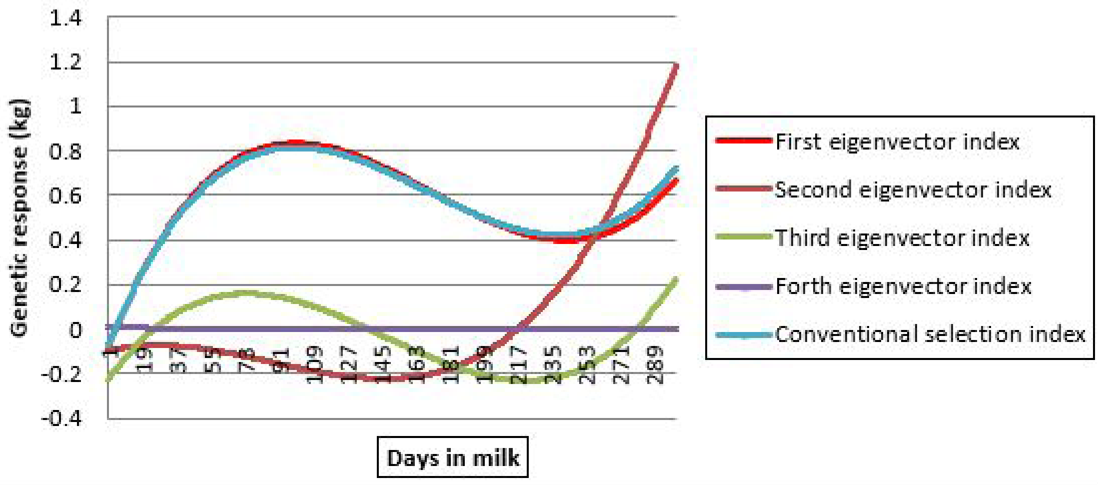

Figure 2. Selection genetic response in daily milk yield based on various individual eigenvector selection indices relative to the conventional selection index.

for the fourth eigenvector (b_4) that was 48.42 kg when the selection was directed for milk yield alone, but it was decreased to be 37.16 kg when the selection was for milk yield and persistency with a_1:a_2 = 1:25. Therefore, as the economic weight of persistence (a2) increases, the index coefficient associated with the second eigenvector increases, again confirming that the second eigenvector is responsible for persistency (Togashi and Lin, 2006; Savegnago *et al*., 2013).

Selection responses using sequential eigenvector indices

If the breeding goal is to improve milk yield only, the genetic response of each of the sequential eigenvector indices (I4, I3, I2, I1) constructed by omitting one eigenvector each is approximately the same, suggesting that the additional genetic benefit from adding additional eigenvectors to the first (major) eigenvector index is minimal. The index coefficient of (b_1 = 200.91) the first eigenvector is much larger than the index coefficients of any of the other 3 eigenvectors (b_2 = 20.80, b_3 = -46.72 and b_4 = 48.42) as stated by Togashi and Lin (2006); Savegnago *et al.* (2013). The genetic gain in milk yield using the sequential eigenvector indices with 1, 2, 3 or 4 eigenvectors (I_1, I_2, I_3, or I_4 respectively) for improving milk yield alone was almost the same as the traditional selection index, EBV was about 171.6 kg in both (Table 4), but a little less gain in indices with milk yield and persistency with relative economic values of 1:25, with the gain ranged from 168.9 to 170.2 kg for milk yield and from -0.028 to 0.045 kg for persistency (Togashi and Lin, 2006; Savegnago *et al*., 2013).

If the purpose of breeding is to improve milk production and persistency, the genetic gain in milk yield using the first sequential eigenvector index was like the traditional selection index (about 171 kg). However, when the second, third and fourth eigenvector indices were added to the first (I_2, I_3 and I_4 respectively), the tendency for the genetic gains for milk yield showed a little decrease in milk yield and persistency (Table 4).

In general, the tendency of the genetic gains for persistency using the sequential eigenvector indices were decreased (from -0.005 for I_2 to -0.154 kg for I_4) and the tendency for the genetic gains in milk yield was stable in case of using sequential eigenvector index for improving milk yield alone (about of 171 kg). But, if the breeding goal is to improve milk yield and persistency all together, additional genetic gains in persistency and high genetic gain for milk yield could be obtained using the 2nd eigenvector index (I^*_2) with the relative economic values of 1:25 as reported by Togashi and Lin (2006); Savegnago *et al.* (2013).

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