

## THE ANALOGY OF REAL OPTIONS TO FINANCIAL OPTIONS

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### ABSTRACT

The real options literature has demonstrated the analogy between decisions concerning real investment opportunities and those concerning financial options. The role of the underlying uncertainty, irreversibility, and time flexibility are very similar in both cases. For a decade immediately after the appearance of the option pricing model, developed by Fischer Black, Robert Merton, and Myron Scholes in 1973, its idea had fascinated a number of researchers and there has been a number of studies using the option pricing technique to solve the valuation problems of various financial instruments such as debentures, convertible bonds, warrants, stocks, and insurance contracts. In the 1980s, the applications of option pricing started to expand beyond the limit of financial instruments to include some economic problems that have the option-like structure. Using the new methods and techniques to find the solutions of the Black and Scholes' partial differential equation, this paper presents the analogy of the real options to the financial options.

**Keywords:** Financial Options, Real Options, Investment, Irreversibility, Uncertainty

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## Introduction

Though the original formula of Black and Scholes was only developed for a simple traded European call option, with simple modification, it was applicable to the valuation of various different types of options. In the 1980s, the applications of option pricing started to expand beyond the limit of financial instruments to include some economic problems that have the option-like structure. Cukierman (1980) was the first to attempt to incorporate the opportunity cost of the waiting option to invest into the valuation of investment projects using a Bayesian framework. Bernanke (1983) introduced the option value of waiting through the "bad news principle". Brennan and Schwartz (1985), McDonald and Siegel (1985), and McDonald and Siegel (1986) formalised the methods and techniques, providing the basis for the research on real options.

The significant development in the area has solved a number of problems related to corporate investment decisions, including, for example, irreversible investment (option to defer) (Dixit, 1992; McDonald and Siegel, 1986); flexibility investment (option to expand, to contract, to shut down and restart) (McDonald and Siegel, 1985; Pindyck, 1988); sequential or staged investment (time to build option) (Majd and Pindyck, 1987; Trigeorgis, 1993); etc. In the first section, the classic financial option pricing formula of Black, Merton, and Scholes will be re-established with different methods and techniques to arrive at the final solutions. A real options valuation model of a simple investment problem is reintroduced so that the analogy between financial options and real options can be demonstrated.

## The Black, Merton, and Scholes Model of Financial Options (1973)

Black initiated the fundamental partial differential equation of the option pricing model in late 1969. Black together with Scholes and Merton complete the Black-Scholes formula for pricing a European call option on an underlying non dividend paying stock whose price is log-normally distributed (Black and Scholes, 1973). In a frictionless efficient market, a portfolio of a long position in a call option to buy the stock and a proportional short position in the stock is made and adjusted continuously to maintain the riskless position. By the law of one price or no-arbitrage, the portfolio then must have the same payoffs as the riskless asset. Suppose the value of a stock,  $P_t$ , evolves according to a geometric Brownian motion of the form:

$$dP_t = \mu P_t dt + \sigma P_t dz \quad (\text{at } t=0, P_0 > 0) \quad (1)$$

where  $\mu$  is the growth rate parameter,  $\sigma$  is the instantaneous standard deviation of the stock value,

$dz$  is the increment of the standard Wiener process<sup>1</sup>. We call  $C(P_t, t)$  the value of the call option to buy the stock and use Ito's lemma to expand call  $C(P_t, t)$ , which is the increment of call  $C(P_t, t)$  over an infinitesimally small time interval  $dt$ :

$$dC = \frac{\partial C}{\partial P_t} dP_t + \frac{1}{2} \frac{\partial^2 C}{(\partial P_t)^2} (dP_t)^2 + \frac{\partial C}{\partial t} dt \quad (2)$$

hence

$$dC = \frac{\partial C}{\partial P_t} dP_t + \frac{1}{2} \sigma^2 P_t^2 \frac{\partial^2 C}{(\partial P_t)^2} dt + \frac{\partial C}{\partial t} dt \quad (3)$$

To offset the resulting unknown rise or fall of  $C(P_t, t)$  as  $P_t$  changes over each time increment  $dt$ , we create a portfolio,  $\Pi$ , consisting of a call option to buy the stock and a short position of  $P_t$  with the amount that equals  $\frac{\partial C}{\partial P_t} P_t$ , and continuously adjust the short sale amount to maintain

the *delta hedge*<sup>2</sup> position. So that the total change in the value of our portfolio,  $d\Pi$  is:

$$d\Pi = dC - \frac{\partial C}{\partial P_t} dP_t = \frac{\partial C}{\partial P_t} dP_t + \frac{1}{2} \sigma^2 P_t^2 \frac{\partial^2 C}{\partial P_t^2} dt + \frac{\partial C}{\partial t} dt - \frac{\partial C}{\partial P_t} dP_t \quad (4)$$

hence

$$d\Pi = dC - \frac{\partial C}{\partial P_t} dP_t = \frac{1}{2} \sigma^2 P_t^2 \frac{\partial^2 C}{\partial P_t^2} dt + \frac{\partial C}{\partial t} dt \quad (5)$$

To avoid arbitrage possibility, the portfolio should only earn the normal interest,  $r$ . Thus, over the time increment  $dt$ , the portfolio earns  $r\Pi dt$ , and:

$$r\Pi dt = rCdt - r \frac{\partial C}{\partial P_t} P_t dt = \frac{1}{2} \sigma^2 P_t^2 \frac{\partial^2 C}{\partial P_t^2} dt + \frac{\partial C}{\partial t} dt \quad (6)$$

Dividing all terms by  $dt$  and rearrange the equation, we arrive at the fundamental *Black-Scholes partial differential equation* for option valuation:

$$rC = rP \frac{\partial C}{\partial P_t} + \frac{1}{2} \sigma^2 P_t^2 \frac{\partial^2 C}{\partial P_t^2} + \frac{\partial C}{\partial t} \quad (7)$$

Notice that there is no term in the Black-Scholes equation which states the exercise price  $K$  of a call option, or its expiry date  $T$ . The Black-Scholes equation can apply to every economic function of the random walk variable  $P_t$ .

To arrive at an analytically tractable solution, some boundary conditions are needed. Since the call option is the right to buy the underlying stock, its current value cannot be higher than the current stock price. At maturity, the value of the call option must not be less than the difference between the stock price and the exercise price. The longer time to maturity, the higher the value of the call option. Black and Scholes applied the heat transfer formula in physics<sup>3</sup> to get the analytical solution for the partial differential equation (7). We present an alternative approach to get the same solution as follows.

Given:

<sup>1</sup> That means  $z(t)$  follows a Markov process with independent increments  $dz = \varepsilon_t \sqrt{dt}$  where  $\varepsilon_t \sim N(0,1)$ , and  $E(\varepsilon_t \varepsilon_s) = 0$  for  $t \neq s$ .  $E(dz) = 0$  and  $\text{Var}(dz) = E(dz)^2 = dt$ .

<sup>2</sup> We call "delta hedge" because the amount of the stock that is sold short is proportional to  $C_\delta = dC/dP_t$ .

<sup>3</sup> The stochastic movement of the stock price in financial markets can be compared with the stochastic movement of heat from a higher temperature place to a lower one, i.e. the stochastic diffusion of prices of stock or other financial securities are very similar to the diffusion of heat. A formal proof of the final solution is given in the Appendix.

$$C(P_t) = \max E[(P_T - K) e^{-rT}, 0] \quad (8)$$

Because  $P_t$  follows a geometric Brownian diffusion process, using Ito's lemma we can obtain:

$$d \ln P_t = \left(r - \frac{1}{2} \sigma^2\right) dt + \sigma dz \quad (9)$$

Thus,  $P_\tau = P_t e^{(r - \frac{1}{2} \sigma^2)(\tau - t) + \sigma z}$ , (with  $\tau \geq t$ ) so that the marginal distribution for  $P_\tau$  is given by  $P_t$  times the natural exponential of a normal variable  $x$ ,  $\mathcal{N}\left(r - \frac{1}{2} \sigma^2\right)(T - t), \sigma^2(T - t)$ . Without loss of generality, we can set  $t=0$  to ease the denotation, then:

$$x \sim \mathcal{N}\left(r - \frac{1}{2} \sigma^2\right)T, \sigma^2 T) \quad (10)$$

and (8) gives

$$C(P_t) = e^{-rT} \int_{\ln(K/P_t)}^{\infty} (P_t e^x - K) \frac{1}{\sqrt{2\pi\sigma^2 T}} \exp\left(-\frac{(x - rT + \frac{1}{2}\sigma^2 T)^2}{2\sigma^2 T}\right) dx \quad (11)$$

$$= \int_{\ln(K/P_t)}^{\infty} P_t e^{x-rT} \frac{1}{\sqrt{2\pi\sigma^2 T}} \exp\left(-\frac{(x - rT + \frac{1}{2}\sigma^2 T)^2}{2\sigma^2 T}\right) dx \quad (12)$$

$$- \int_{\ln(K/P_t)}^{\infty} K e^{-rT} \frac{1}{\sqrt{2\pi\sigma^2 T}} \exp\left(-\frac{(x - rT + \frac{1}{2}\sigma^2 T)^2}{2\sigma^2 T}\right) dx$$

$$= P_t \int_{\ln(K/P_t)}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2 T}} \exp\left(-\frac{(x - rT + \frac{1}{2}\sigma^2 T)^2 + 4(x - rT)\frac{1}{2}\sigma^2 T}{2\sigma^2 T}\right) dx \quad (13)$$

$$- K e^{-rT} \int_{\ln(K/P_t)}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2 T}} \exp\left(x - \frac{(x - rT + \frac{1}{2}\sigma^2 T)^2}{2\sigma^2 T}\right) dx$$

$$= P_t \int_{\ln(K/P_t)}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x - rT - \frac{1}{2}\sigma^2 T)^2}{2\sigma^2 T}\right) d\frac{x}{\sigma\sqrt{T}} \quad (14)$$

$$- K e^{-rT} \int_{\ln(K/P_t)}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x - rT + \frac{1}{2}\sigma^2 T)^2}{2\sigma^2 T}\right) d\frac{x}{\sigma\sqrt{T}}$$

If we let  $d_1 = \frac{x - (r + \frac{1}{2} \sigma^2)T}{\sigma\sqrt{T}}$  and  $d_2 = \frac{x - (r - \frac{1}{2} \sigma^2)T}{\sigma\sqrt{T}}$ , and replacing  $d_1$  and  $d_2$  into (14), we

can write:

$$C(P_t) = P_t \int_{(\ln(K/P_t) - rT - \frac{1}{2}\sigma^2 T)/\sigma\sqrt{T}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{d_1^2}{2}\right) d(d_1) \quad (15)$$

$$- K e^{-rT} \int_{(\ln(K/P_t) - rT + \frac{1}{2}\sigma^2 T)/\sigma\sqrt{T}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{d_2^2}{2}\right) d(d_2)$$

By construction, we have a pair of standard cumulative normal distributions,  $d_1 \sim \mathcal{N}(0,1)$  and  $d_2 \sim \mathcal{N}(0,1)$ . Thus the final analytical solution for the partial differential equation (7) can be easily calculated.

$$C(P_t, t) = P_t \Phi(d_1) - K e^{-r(T-t)} \Phi(d_2) \quad (16)$$

$$d_1 = \frac{\ln\left(\frac{P_t}{K}\right) + \left(r + \frac{1}{2} \sigma^2\right)(T - t)}{\sigma\sqrt{T - t}} \quad (17)$$

$$d_2 = \frac{\ln\left(\frac{P_t}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \quad (18)$$

where  $\Phi(\cdot)$  is the cumulative normal density function, and  $T-t$  is the time-to maturity of the option. Equation (16) can be written as  $C(P_t, t) = e^{-r(T-t)} [P_t e^{r(T-t)} \Phi(d_1) - K \Phi(d_2)]$ . Thus,  $\Phi(d_2)$  is the probability that the option will be exercised in a risk neutral world, and  $K\Phi(d_2)$  is the expected future expense at time  $T$  if the option is exercised. The expected future income of the option at time  $T$  is  $P_t e^{r(T-t)} \Phi(d_1)$ . The future income of the option depends on the value of the underlying asset that equals  $P_T$  if  $P_T > 0$  and is zero otherwise. The expected future income net of expected future expense gives the expected option value at time  $T$ , which is then discounted to present value as in (16). The option value, *ceteris paribus*, depends only on the current stock price, exercise price, riskless interest rate, volatility, and time to maturity of the option.

### The Basic Real Options Model of Investment

This section gives a formal treatment of the valuation of the option to defer investment. The methods and techniques for deriving the final solutions to the valuation problem, introduced by McDonald and Siegel (1986) and Dixit and Pindyck (1994) is very much the same as what we have presented for the financial options. Suppose the value of a project,  $V$ , evolves according to a stochastic diffusion process called the geometric Brownian motion of the form:

$$dV = (\rho - \delta)Vdt + \sigma Vdz \quad (19)$$

Where  $\rho$  is the instantaneous actual expected return on the project,  $\delta$  denotes the proportional cash flow pay-out (dividend) on the operating project,  $\sigma$  is the instantaneous standard deviation of the project value, and  $dz$  is the increment of the standard Wiener process. Our purpose is to find out the optimal expected value,  $V^*$ , at which firms should decide to invest and maximise its expected net payoff value of the investment opportunity,  $F(V)$ , over the time horizon,  $T$

The pricing of a real option to invest is very much the same as the pricing of a financial call option to buy a dividend-paying stock. However, because the value of the project,  $V$ , is not traded on financial markets, we assume that the stochastic changes in  $V$  must be spanned by an existing traded asset or a portfolio of assets that is perfectly correlated with  $V$ .

As above,  $F(V)$  is the value of the option to invest and  $V$  is log-normally distributed. Then we expand  $dF$  over a small time increment  $dt$  using Ito's lemma and equation (19):

$$dF = F_V dV + \frac{1}{2} F_{VV} (dV)^2 = F_V dV + \frac{1}{2} \sigma^2 V^2 F_{VV} dt \quad (20)$$

Like the case of a call option to buy a stock, over each short time interval,  $dt$ , we create a portfolio,  $\Pi$ , consisting of a long position in the option to invest of  $F(V)$  and a short position in the project (or an equivalent asset) with the value, that reflects *delta hedge* strategy, of  $F_V V$ . Then the value of the portfolio is  $\Pi = F(V) - F_V V$ .

Unlike the analysis provided by Black and Scholes in their original paper for a financial option to buy a non dividend paying stock, our real option to invest can be considered as a call

option that pays dividends with the continuous rate,  $\delta$ . As  $V$  follows a diffusion process of equation (19), the continuous rate of value appreciation is, which can be considered as capital gains or retained earnings from holding the project. The dividends and capital gains together are the expected returns on the project, which are the expected continuous rate of profit (or total earnings) when the investment is made. In our current portfolio, we are in short position of the project, hence over each time increment,  $dt$ , we must pay out  $\delta F_V V dt$  as dividends or distributed earnings. Thus, over small time increment  $dt$ , the change in value of our portfolio will be

$$d\Pi = dF - F_V dV - \delta F_V V dt \quad (21)$$

Substituting for  $dF$  above gives:

$$d\Pi = F_V dV + \frac{1}{2} \sigma^2 V^2 F_{VV} dt - F_V dV - \delta F_V V dt \quad (22)$$

and

$$d\Pi = \frac{1}{2} \sigma^2 V^2 F_{VV} dt - \delta F_V V dt \quad (23)$$

Over the time increment  $dt$ , to avoid arbitrage possibility, the portfolio should only earn the normal interest rate,  $r$ , and the sum is  $r\Pi dt$ . Thus:

$$d\Pi = r\Pi dt = r(F(V) - F_V V) dt = rF dt - F_V V dt \quad (24)$$

Compare the immediately above two equations (23) and (24), rearrange the terms, and divide the two sides by  $dt$ , we arrive at the final differential equation for the real option to invest:

$$\frac{1}{2} \sigma^2 V^2 F_{VV} dt - \delta F_V V dt = rF dt - rF_V V dt \quad (25)$$

$$\frac{1}{2} \sigma^2 V^2 F_{VV} dt + rF_V V dt - \delta F_V V dt - rF dt = 0 \quad (26)$$

$$\frac{1}{2} \sigma^2 V^2 F_{VV} + (r - \delta) F_V V - rF = 0 \quad (27)$$

This differential equation is very much the same as the fundamental Black-Scholes partial differential equation, except for the fact that it does not include the first order derivative of  $F$  with respect to  $t$  for the current equation is only differentiated with respect to a single variable,  $V$ , and it includes a term related to the paid-out dividends to reflect the changes in the structure of the option.

### The Analogy of Real Options to Financial Options to Invest

Investment *irreversibility*, *uncertainty*, and *timing flexibility* give rise to the option-like feature of investment opportunities. By definition, a financial option is a contract that gives the holder the right but not the obligation to buy or sell a certain underlying asset at the price agreed today (called exercise price or strike price) for delivery on a given date in the future (called exercise date or expiry date). A call option offers the call holder the right to buy, while a put option gives the put holder the right to sell. An American option permits the holder to exercise the option on or before the expiry date, while a European option allows the holder to exercise his/her right only on the date of expiry. For example, an American call holder of an IBM stock will be able to exercise his/her right and buy that stock any time on or before the expiry date at today's agreed price. Analogous to the financial option, an investment opportunity is an American call option with a

perpetual expiry date. To put it differently, the investor has the right to pay the exercise price (which, in this case, is the cost of investment and is irreversible) and receive an asset in return (which is the realised value of the investment) at any time on or before the expiry date, which is perpetual.

As we have observed in our valuations of financial and real options, in a world where the law of one price prevails, arbitrage will govern the price of the options regardless of their future actual price. Therefore, options have a value that does not depend on the future actual price of the underlying asset but, *ceteris paribus*, depends only on the current price of the underlying asset. In addition, they are rights and not obligations to buy or sell the underlying asset, their holders can always enjoy the favourable price movement whilst limiting his/her loss when the price moves adversely, and their value is influenced by the uncertainty and time to maturity.

Take the call option in our analysis as an example. At maturity, should price moves favourably, the option will be exercised and give the holder payoff, that is the difference between the actual realised price of the stock and the exercise price,  $P-K$ , which is called the intrinsic value of the option. If the price of the stock moves adversely, the option is not exercised and the payoff is zero. Before maturity, when the price of the underlying stock is at the lower range, there is a possibility that the price will improve in the future and the holder can expect to enjoy the possible payoff, which is the difference between the realised price of the underlying stock and the cost to obtain the stock under the call option contract, the exercise price. He/she can limit the loss when the stock price moves adversely.

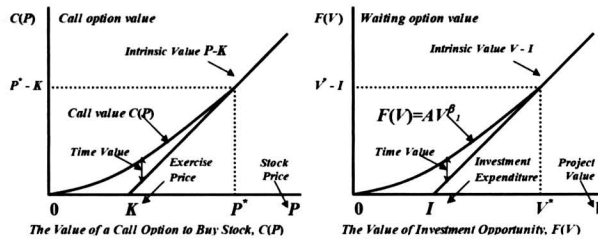


Figure 1 The Analogy of Real Options to Financial Options

The higher the uncertainty of future stock prices and the longer time to maturity, the higher the value of the option,  $C(P)$ , as higher uncertainty and longer time to maturity will increase the possibility of a higher realised price as well as the possibility of a lower realised price. But the holder can enjoy the benefit of upside price improvement while limit the loss of the downside price deterioration, hence he must pay something to enjoy this favourable condition which is the price  $C(P)$  of the option. The difference between the value of the option at the lower range of stock price and the intrinsic value is called the time value of the option. There is a cut-off point  $P^*$ , at which the time value of the option disappears, and the option value coincides with its intrinsic value, as  $P$  is already too high, and waiting is no longer valuable.

Similar to what we have analysed above, in a real option, we have the opportunity to make

an investment at a known cost,  $I$ , which is similar to the exercise price. And the investment would provide us with underlying real assets, which are factories or equipment.

If the investment opportunity is a now or never opportunity, then it is like the option at maturity; the firm has no choice but to make the investment decision should the expected present value,  $V$ , of future possible cash flows be higher than the exercise price, which is the investment cost,  $I$ , then the firm should invest immediately. Firms are not required to invest if the expected present value of future profits is less than the exercise price and their "loss" is limited to zero.

However, in many cases, the investment opportunity does not disappear immediately and firms can choose to invest either now or later. In those cases, firms will often choose to wait for new information to arrive so that they can make the investment decision later when market conditions improve to ensure the profitability of the project. At the same time, they can limit their loss by not investing when the market conditions deteriorate. Hence, under uncertainty, waiting has a value which is similar to the time value of the option and firms only invest when the expected return covers both the intrinsic value of the investment project and its time value.

## Conclusions

The real option to invest is in many ways similar to the financial call option. However, unlike financial options where value is derived from other financial instruments, the option to invest can be considered a real option as it gives the right to obtain a real asset. Once investment is carried out, the option is gone, so the option value is an opportunity cost of investing. Managers must add this opportunity cost into its full cost of investment, hence, the hurdle level increases. The higher the uncertainty, the higher the hurdle level is. This is an important implication for real options research on corporate investment decisions.

The real options approach to investment decisions under uncertainty gives us some important insight into firms' investment behaviour. First, the lost option value as firms decide to exercise their waiting option to invest is an opportunity cost that must be included as part of the cost of investment. Thus, the traditional NPV rule must be modified to include this opportunity cost. Second, this opportunity cost is sensitive to uncertainty or changing economic conditions over the future value of the investment. Hence uncertainty has an important impact on investment spending and sometimes this impact is even more important than interest rates. Third, firms would make (or abandon) an investment only when the present value of expected return of the project reaches a "hurdle" level, which is sufficiently higher (or lower) than the cost of capital. The hurdle level is an increasing function of uncertainty.

This insight has implications for firm managers in preparing their capital budgeting and carrying out a series of option-like managerial operations, as well as for policymakers in issuing policies to induce investments.



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